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NONSTATIONARY HEAT TRANSFER IN AN INHOMOGENEOUS THERMALLY INSULATED TIMBER SAMPLE

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A mathematical model and numerical methods to calculate the thermal state of a fragment of the exterior timber wall of a building are suggested. The character of the distribution of temperature fields in homogeneous and inhomogeneous (thermally insulated) timbers and the influence of the thermophysical and geometrical characteristics of a homogeneous timber sample and warmth-keeping lagging on it are determined.

According to sanitation and hygiene standards, wood is among the most suitable building materials; therefore its use in residential buildings is most expedient.

The current requirements for the heat-reflecting properties of enclosing structures of buildings [1] prevent the use of homogeneous exterior timber walls in building under cold climatic conditions. The use of inhomogeneous timbers with a longitudinal axial hole filled with effective warmth-keeping lagging as an element of the enclosing structure [2] improves the thermotechnical characteristics of exterior timber walls of buildings.

In this connection, of scientific and practical interest are a theoretical investigation of the laws governing the process of heat transfer in inhomogeneous, thermally insulated timbers and substantiation of the technique for increasing the efficiency of the heat-reflecting properties of exterior timber walls that was suggested in [2].

Physicomathematical Statement of the Problem. We investigate heat transfer through a plane inhomogeneous system consisting of timber sample 1 with an axial hole 2 filled with warmth-keeping lagging (Fig. 1). The timber sample and lagging have the shape of straight parallelepipeds, the cross sections of which form squares with sides d_1 and d_2, respectively. The thermophysical characteristics (λ_i, ρ_i, c_i, i = 1, 2) of the materials of the system, its geometrical dimensions, the temperature of the external ($T_{g,e}$) and inside ($T_{g,ins}$) media, and the coefficients of heat transfer on the outer (α_w) and inner (α_0) enclosure surfaces, as well as the radiative parameters of the outer surface of the enclosure (ε_w) and the external medium (ε_e) are known. It is necessary to calculate temperature fields in the cross section of an inhomogeneous timber sample and heat fluxes through its inner and outer surfaces.

Heat transfer in the cross section of the inhomogeneous timber sample in regions 1 and 2 is described by two-dimensional, nonlinear, nonstationary heat-conduction equations:

$$(\rho c)_i \frac{\partial T_i}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda_i \frac{\partial T_i}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_i \frac{\partial T_i}{\partial y}\right), \quad i = 1, 2 . \tag{1}$$

The system of equations (1) is closed by the following initial and boundary conditions:

$$T\Big|_{\tau=0} = T_{in}(x, y) ; \tag{2}$$

$$-\lambda_1 \frac{\partial T_1}{\partial x}\Big|_{x=0} = \alpha_0 (T_{g,ins} - T_0) ; \tag{3}$$

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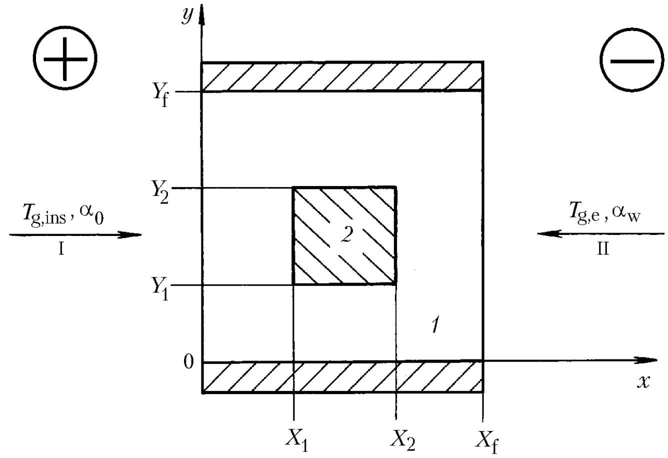


Fig. 1. Cross section of a thermally insulated timber sample: 1) wood; 2) warmth-keeping lagging; I) convective heat transfer; II) radiative-convective heat transfer.

$$\lambda_1 \frac{\partial T_1}{\partial x}\bigg|_{x=X_f} = \alpha_w (T_{g,e} - T_w) + \sigma\varepsilon_{eff} (T_{g,e}^4 - T_w^4) ; \tag{4}$$

$$\frac{\partial T_1}{\partial y}\bigg|_{y=0} = 0 ; \tag{5}$$

$$\frac{\partial T_1}{\partial y}\bigg|_{y=Y_f} = 0 ; \tag{6}$$

$$T_1\big|_{x=X_1} = T_2\big|_{x=X_1} , \quad \lambda_1 \frac{\partial T_1}{\partial x}\bigg|_{x=X_1} = \lambda_2 \frac{\partial T_2}{\partial x}\bigg|_{x=X_1} , \quad Y_1 \le y \le Y_2 ; \tag{7}$$

$$T_1\big|_{x=X_2} = T_2\big|_{x=X_2} , \quad \lambda_1 \frac{\partial T_1}{\partial x}\bigg|_{x=X_2} = \lambda_2 \frac{\partial T_2}{\partial x}\bigg|_{x=X_2} , \quad Y_1 \le y \le Y_2 ; \tag{8}$$

$$T_1\big|_{y=Y_1} = T_2\big|_{y=Y_1} , \quad \lambda_1 \frac{\partial T_1}{\partial y}\bigg|_{y=Y_1} = \lambda_2 \frac{\partial T_2}{\partial y}\bigg|_{y=Y_1} , \quad X_1 \le x \le X_2 ; \tag{9}$$

$$T_1\big|_{y=Y_2} = T_2\big|_{y=Y_2} , \quad \lambda_1 \frac{\partial T_1}{\partial y}\bigg|_{y=Y_2} = \lambda_2 \frac{\partial T_2}{\partial y}\bigg|_{y=Y_2} , \quad X_1 \le x \le X_2 . \tag{10}$$

On the boundaries of the computational domain, the following conditions are set: at $x = 0$, the condition of convective heat transfer (3); at $x = X_f$, the condition of radiative-convective heat transfer (4); at $y = 0$ and $y = Y_f$, adiabatic conditions (5) and (6), and on the inner boundaries of the system — conditions of the fourth kind (7)–(10). The function ε_{ef} is calculated from the Christiansen formula $\varepsilon_{ef} = (\varepsilon_e^{-1} + \varepsilon_w^{-1} - 1)^{-1}$.

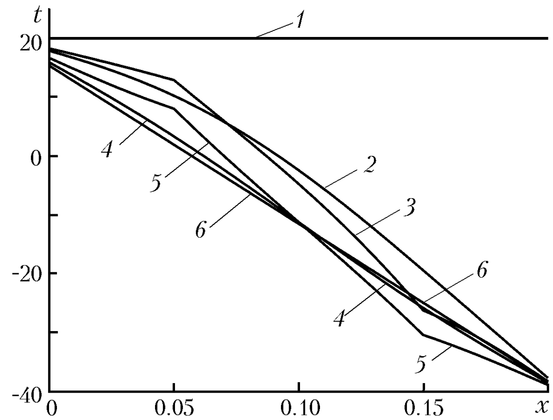


Fig. 2. Dependence of temperature on x at $y = Y_f$ (2, 4) and $y = Y_f/2$ (3, 5) for thermally insulated (1–5) and homogeneous (6) timbers at different time instants τ: 1) 0; 2, 3) 12; 4–6) 168 h. x, m; t, ^oC.

Method of Problem Solution and Results of Numerical Calculations. For numerical solution of the problem, the method of splitting is used [3]. The resulting one-dimensional equations of heat conduction in one- and three-layer regions in the x and y directions were calculated by the iterative-interpolation method of [4] with iterations with respect to the coefficients with the accuracy assigned.

A numerical solution of the problem using the algorithm given above is performed using a program developed on the module principle and coded in FORTRAN for PC. Calculations of the thermal state of the thermally insulated timber sample were carried out with the following initial data [(1)–(4) are the numbers of variants]:

(1) $d_1 = 0.2$ m, $\lambda_1 = 0.15$ W/(m·K), $c_1 = 2300$ J/(kg·K), $\rho_1 = 500$ kg/m^3;

(2) $d_1 = 0.2$ m, $\lambda_1 = 0.3$ W/(m·K), $c_1 = 2300$ J/(kg·K), $\rho_1 = 900$ kg/m^3;

(3) $d_1 = 0.26$ m, $\lambda_1 = 0.15$ W/(m·K), $c_1 = 2300$ J/(kg·K), $\rho_1 = 500$ kg/m^3;

(4) $d_1 = 0.26$ m, $\lambda_1 = 0.3$ W/(m·K), $c_1 = 2300$ J/(kg·K), $\rho_1 = 900$ kg/m^3;

(1)–(4) $\lambda_2 = 0.05$ W/(m·K), $c_2 = 1470$ J/(kg·K), $\rho_2 = 60$ kg/m^3, $d_2 = 0.1$.

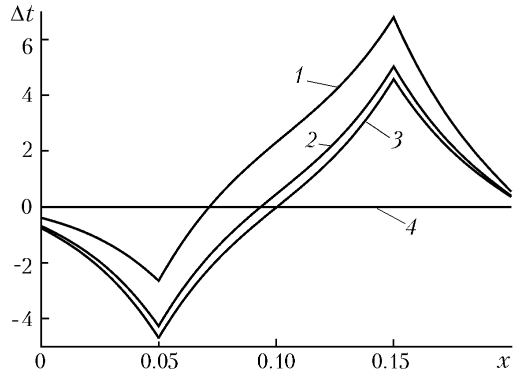
An analysis of the results will be made in degrees Centigrade.

Figure 2 shows the temperature field in the thermally insulated timber sample calculated for the initial data of variant (1). An analysis of this figure shows that the temperature profiles in the planes $y = Y_f$ (curves 2 and 4) and $y = Y_f/2$ (curves 3 and 5) are different, which is due to the presence of a low-conducting insert in the timber sample. On decrease in the external air temperature from 20 to –40^{o}C, an abrupt change in the temperature profile over the timber-sample thickness is observed. During the first 12 hours from the beginning of the fall in temperature, the temperature on the outer surface of the timber sample undergoes marked changes: from 20^{o}C to –36.6^{o}C at $y = Y_f$ (curve 2) and to –38.2^{o}C at $y = Y_f/2$ (curve 3). The temperature on the inner surface of the timber sample undergoes little change during this time and is equal to about 17.9^{o}C at $y = Y_f$ and to 18.2^{o}C at $y = Y_f/2$. On reaching a steady-state regime of heat conduction, the temperature on the outer surface of the timber sample becomes equal to about –38.4^{o}C at $y = Y_f$ (curve 4) and –38.8^{o}C at $y = Y_f/2$ (curve 5). On the inner surface of the timber sample, the values of the temperature are about 15.9^{o}C at $y = Y_f$ and 16.7^{o}C at $y = Y_f/2$.

The foregoing results of calculations show that on establishment of the steady-state conditions of heat conduction the temperature on the inner surface of the timber sample at $y = Y_f/2$ is about 4.8% higher than that at $y = Y_f$.

For other variants of calculations, the similar temperature profiles coincide qualitatively but somewhat disagree quantitatively.

The calculation results for variant (2) show that in comparison with variant (1) the temperature of the timber sample increases on the outer surface and decreases on the inner surface. Here, the temperature difference on the outer surface of the timber sample for variants (1) and (2) in a steady-state regime of heat conduction is from 1.8 to 2.9% and on the inner surface — from 12 to 17.6%. Thus, the temperature on the outer surface of the timber sample for variant (2) is equal to –37.3^{o}C at $y = Y_f$ and to –38.1^{o}C at $y = Y_f/2$. On the inner surface of the timber sample, the temperature values are 13.1^{o}C at $y = Y_f$ and 14.7^{o}C at $y = Y_f/2$, respectively.



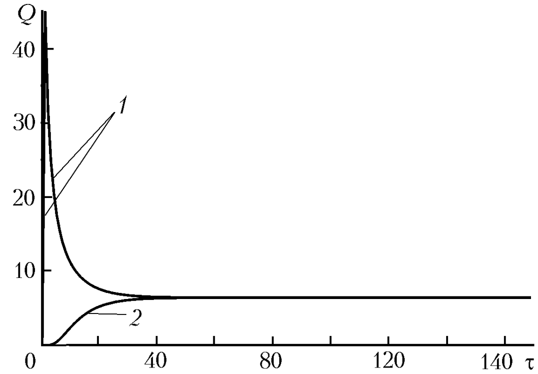


Fig. 3. Distribution of temperature differences over x between the temperatures on the periphery $y = Y_f$ and axis $y = Y_f/2$ for thermally insulated (1–3) and homogeneous (4) timbers at different time instants τ: 1) 12; 2) 24; 3, 4) 168 h. x, m; t, ^oC.

Fig. 4. Heat fluxes through the exterior (1) and interior (2) surfaces of the thermally insulated timber sample vs. time. t, h; Q, W.

The calculation results for variants (3) and (4) show that when a steady-state regime of heat conduction is established, the difference between the temperature on the outer surface of the timber sample is 2.0–2.6% and on the inner surface it is 12.4–15.7%. Thus, the temperature on the outer surface of the timber sample for variant (3) is –38.7^{o}C at $y = Y_f$ and –38.9^{o}C at $y = Y_f/2$, whereas on the inner surface it is equal to 16.6^{o}C at $y = Y_f$ and 17.0^{o}C at $y = Y_f/2$. For variant (4) these values are: –37.7^{o}C at $y = Y_f$ and –38.1^{o}C at $y = Y_f/2$ for the outer surface and 14.0^{o}C at $y = Y_f$ and 14.9^{o}C at $y = Y_f/2$ for the inner surface.

Figure 3 shows the distributions of temperature differences over x between the temperatures on the periphery $y = Y_f$ and axis $y = Y_f/2$ for thermally insulated and homogeneous timbers at different moments of time; they were obtained using the initial data of variant (1).

During the first 12 hours from the beginning of the abrupt fall in the temperature of the external air from 20 to –40^{o}C, an intensive cooling of the outer part of the timber sample occurs (curve 1). Here, the maximum temperature difference for variants (1) and (2) at $x = 0.15$ m is 6.8 and 7.4^{o}C, respectively, and at $x = 0.05$ m it is equal to –2.7 and –5.9^{o}C. For variants (3) and (4), the maximum temperature difference at $x = 0.18$ m is equal to 7.5 and 8.3^{o}C, respectively, and at $x = 0.05$ m it is –1.1 and –4.7^{o}C.

When the system attains a steady-state regime of heat conduction (curve 3), the temperature differences in the timber-sample sections considered are equalized, and for variants (1)–(4) they are equal to about ±4.6, ±6.6, ±4.4, and ±6.3^{o}C.

Thus, it has been established that for all variants of calculation the maximum perturbations of the temperature field are observed in the zones where the lagging contacts with the wood. In the steady-state mode of heat transfer in the center of timbers at $x = 0.1$ m [variants (1) and (2)] and at $x = 0.13$ m [variants (3) and (4)], there is a section with a maximum value of transmission heat, before which the heat from the axis of the timber sample is removed to its periphery and after which, on the contrary, heat is supplied from the periphery to the timber-sample axis.

Figure 4 presents the dependences of heat fluxes on time that were calculated for variant (1).

Upon a sharp decrease in the external temperature from 20 to –40^{o}C, during the first hour of cooling, an intense 45-W efflux of heat from the outer surface of the timber sample (curve 1) occurs. After the construction reaches a steady-state regime of heat conduction, the powers of the heat fluxes through the outer and inner timber-sample surfaces become equal and amount to 6.4 W for the thermally insulated timber sample and 8 W for the homogeneous one. For variant (2), the maximum efflux of heat from its outer surface is equal to 60.4 W. After the construction attains a steady-state regime of heat conduction, the heat losses amount to 10.6 W for the thermally insulated timber-sample and 14.5 W for the homogeneous one. For variants (3) and (4), the values of heat losses on rapid decrease in the outer air also reach a maximum during the first hour of cooling and are equal to 58.6 and 80

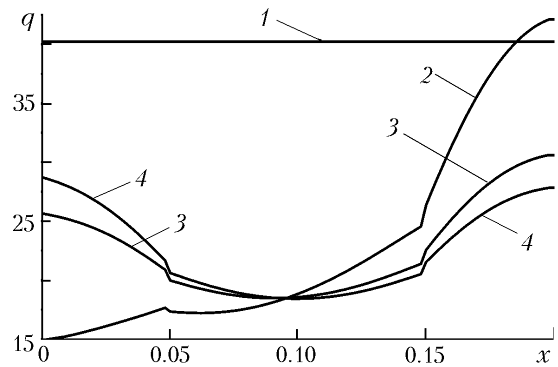


Fig. 5. Distribution of heat-flux densities over x in the section $y = Y_f/2$ for homogeneous (1) and thermally insulated (2–4) timbers at different time instants τ: 1, 4) 168; 2) 12; 3) 24 h. x, m; q, W/m^2.

W. On attainment of a steady-state regime of heat transfer, the heat losses through the thermally insulated and homogeneous timber samples are 7.2 and 8.2 W for variant (3) and 12.6 and 15.2 W for variant (4). The time of emergence into the steady-state regime of heat conduction for the constructions investigated varies from 30 h [variant (2)] to 110 h [variant (3)].

Figure 5 depicts the distributions of the densities of heat fluxes over x in the section $Y_f/2$ at different time instants calculated for variant (1).

The value of the density of the heat flux q in a steady-state regime of heat conduction for the homogeneous timber sample is about 40.2 W/m^2 for variant (1), 72.7 W/m^2 for variant (2), 31.7 W/m^2 for variant (3), and 58.5 W/m^2 for variant (4). The curves of the distributions of heat-flux densities in the homogeneous (curve 1) and thermally insulated (curve 4) timber samples in a steady-state regime of heat conduction differ substantially. The densities of heat fluxes through the thermally insulated timber sample have minimums at the center of the timber sample, the values of which for variants (1)–(4) are equal to 18.4, 19.2, 15.0, and 16.0 W/m^2, respectively. The heat-flux densities on the outer and inner surfaces of the timber sample for the calculation variants considered are practically equal and amount to 28.7, 45.9, 25.9, and 44.3 W/m^2.

Thus, the foregoing numerical investigation of the thermal state of homogeneous and inhomogeneous timbers has revealed certain regularities in the distribution of temperature fields and heat-flux densities for different thermophysical and geometrical characteristics of timbers. The results obtained theoretically substantiate the method [2] of increasing the heat-protecting properties of exterior timber walls. The numerical technology developed allows one to predict the thermal state of exterior timber walls under cold climatic conditions and more rationally tackle the problem of selecting the systems for extra warmth-keeping.

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NOTATION

c, specific heat, J/(kg·K); d_1, width of a timber sample, m; d_2, width of warmth-keeping lagging, m; q, heat-flux density, W/m^2; Q heat-flux; W; t, temperature, ^{o}C; T, absolute temperature, K; Δt, temperature difference on the periphery ($y = Y_f$) and axis ($y = Y_f/2$) of the timber sample, ^{o}C; x, y, axes of Cartesian coordinate system, m; X_i (i = 1, 2) and Y_i (i = 1, 2), x- and y-coordinates of the inner boundaries of computational subdomains, m; α, heat-transfer coefficient, W/(m^2·K); ε, emissivity, λ, thermal conductivity, W/(m·K); ρ, density, kg/m^3; σ, Stefan–Boltzmann constant, W/(m^2·K^4); τ, time, h. Subscripts and superscripts: e, external medium; ef, effective; f, finite value; g, air; i, number of computational domain; in, initial state; ins, inside medium; w, outer surface of enclosure; 0, inner surface of enclosure; 1, wood; 2, warmth-keeping lagging.

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